10.3: 2, 12, 40

2. $\mathbf{r}'(t) = \langle 2t, \cos t + t\sin t - \cos t, -\sin t + t\cos t + \sin t\rangle = \langle 2t, t\sin t, t\cos t\rangle \Rightarrow$
$|\mathbf{r}'(t)| = \sqrt{(2t)^2 + (t\sin t)^2 + (t\cos t)^2} = \sqrt{4t^2 + t^2(\sin^2 t + \cos^2 t)} = \sqrt{5}\,|t| = \sqrt{5}\,t$ for $0 \le t \le \pi$. Then using Formula 3, we have $L = \int_0^{\pi} |\mathbf{r}'(t)|\,dt = \int_0^{\pi} \sqrt{5}\,t\,dt = \sqrt{5}\,\frac{t^2}{2}\Big]_0^{\pi} = \frac{\sqrt{5}}{2}\pi^2$.

12. (a) $\mathbf{r}'(t) = \langle 2t, t\sin t, t\cos t\rangle \Rightarrow |\mathbf{r}'(t)| = \sqrt{4t^2 + t^2\sin^2 t + t^2\cos^2 t} = \sqrt{5t^2} = \sqrt{5}\,t$ (since $t > 0$). Then
$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \frac{1}{\sqrt{5}\,t}\langle 2t, t\sin t, t\cos t\rangle = \frac{1}{\sqrt{5}}\langle 2, \sin t, \cos t\rangle$. $\mathbf{T}'(t) = \frac{1}{\sqrt{5}}\langle 0, \cos t, -\sin t\rangle \Rightarrow$
$|\mathbf{T}'(t)| = \frac{1}{\sqrt{5}}\sqrt{0 + \cos^2 t + \sin^2 t} = \frac{1}{\sqrt{5}}$. Thus $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} = \frac{1/\sqrt{5}}{1/\sqrt{5}}\langle 0, \cos t, -\sin t\rangle = \langle 0, \cos t, -\sin t\rangle$.

(b) $\kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{1/\sqrt{5}}{\sqrt{5}\,t} = \frac{1}{5t}$.

40. $t = 1$ at $(1, 1, 1)$. $\mathbf{r}'(t) = \langle 1, 2t, 3t^2\rangle$. $\mathbf{r}'(1) = \langle 1, 2, 3\rangle$ is normal to the normal plane, so an equation for this plane is $1(x-1) + 2(y-1) + 3(z-1) = 0$, or $x + 2y + 3z = 6$.

$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \frac{1}{\sqrt{1 + 4t^2 + 9t^4}}\langle 1, 2t, 3t^2\rangle$. Using the product rule on each term of $\mathbf{T}(t)$ gives

$$\mathbf{T}'(t) = \frac{1}{(1 + 4t^2 + 9t^4)^{3/2}}\left\langle -\tfrac{1}{2}(8t + 36t^3), 2(1 + 4t^2 + 9t^4) - \tfrac{1}{2}(8t + 36t^3)2t, 6t(1 + 4t^2 + 9t^4) - \tfrac{1}{2}(8t + 36t^3)3t^2\right\rangle$$

$$= \frac{1}{(1 + 4t^2 + 9t^4)^{3/2}}\langle -4t - 18t^3, 2 - 18t^4, 6t + 12t^3\rangle = \frac{-2}{(14)^{3/2}}\langle 11, 8, -9\rangle \text{ when } t = 1.$$

$\mathbf{N}(1) \parallel \mathbf{T}'(1) \parallel \langle 11, 8, -9\rangle$ and $\mathbf{T}(1) \parallel \mathbf{r}'(1) = \langle 1, 2, 3\rangle \Rightarrow$ a normal vector to the osculating plane is $\langle 11, 8, -9\rangle \times \langle 1, 2, 3\rangle = \langle 42, -42, 14\rangle$ or equivalently $\langle 3, -3, 1\rangle$. An equation for the plane is $3(x-1) - 3(y-1) + (z-1) = 0$ or $3x - 3y + z = 1$.

10.4: 22; [One More Problem]

22. As in Exercise 21, $\mathbf{v}(t) = 250\sqrt{3}\,\mathbf{i} + (250 - gt)\,\mathbf{j}$ and $\mathbf{r}(t) = 250\sqrt{3}\,t\,\mathbf{i} + \left(250t - \frac{1}{2}gt^2\right)\mathbf{j} + \mathbf{c}_2$. But $\mathbf{r}(0) = 200\,\mathbf{j}$, so $\mathbf{c}_2 = 200\,\mathbf{j}$ and $\mathbf{r}(t) = 250\sqrt{3}\,t\,\mathbf{i} + \left(200 + 250t - \frac{1}{2}gt^2\right)\mathbf{j}$.

(a) $200 + 250t - \frac{1}{2}gt^2 = 0$ implies that $gt^2 - 500t - 400 = 0$ or $t = \frac{500 \pm \sqrt{500^2 + 1600g}}{2g}$. Taking the positive t-value gives $t = \frac{500 + \sqrt{250{,}000 + 1600g}}{2g} \approx 51.8$ s. Thus the range is $\left(250\sqrt{3}\right)\frac{500 + \sqrt{250{,}000 + 1600g}}{2g} \approx 22.4$ km.

(b) $0 = \frac{d}{dt}\left(200 + 250t - \frac{1}{2}gt^2\right) = 250 - gt$ implies that the maximum height is attained when $t = 250/g \approx 25.5$ s and thus the maximum height is $\left[200 + (250)\left(\frac{250}{g}\right) - \frac{g}{2}\left(\frac{250}{g}\right)^2\right] = 200 + \frac{(250)^2}{2g} \approx 3.4$ km.

Alternate solution: Because the projectile is fired in the same direction and with the same velocity as in Exercise 21, but from a point 200 m higher, the maximum height reached is 200 m higher than that found in Exercise 21, that is, 3.2 km + 200 m = 3.4 km.

(c) From part (a), impact occurs at $t = \frac{500 + \sqrt{250{,}000 + 1600g}}{2g}$. Thus the velocity at impact is
$250\sqrt{3}\,\mathbf{i} + \left[250 - g\,\frac{500 + \sqrt{250{,}000 + 1600g}}{2g}\right]\mathbf{j}$, so $|\mathbf{v}| \approx \sqrt{(250)^2(3) + (250 - 51.8g)^2} \approx 504$ m/s.